

Spectrum Mobility Games

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Abstract—Cognitive radio gives users the ability to switch channels and make use of dynamic spectrum opportunities. However, switching channels takes time, and may affect the quality of a user's transmission. When a cognitive radio user's channel becomes unavailable, sometimes it may be better waiting until its current channel becomes available again. Motivated by the recent FCC ruling on TV white space, we consider the scenario where cognitive radio users are given the foreknowledge of channel availabilities. Using this information, each user must decide when and how to switch channels. The users wish to exploit spectrum opportunities, but they must take account of the cost of switching channels and the congestion that comes from sharing channels with one another. We model the scenario as a game which, as we show, is equivalent to a network congestion game in the literature after proper and non-trivial transformations. This allows us to design a protocol which the users can apply to find Nash equilibria in a distributed manner. We further evaluate how the performance of the proposed schemes depends on switching cost using real channel availability measurements.

I. INTRODUCTION

With the unprecedented growth in the number of mobile devices and wireless services, the radio frequency spectrum is becoming increasingly valuable. On one hand, it is extremely difficult and time-consuming to obtain new unallocated spectrum. On the other hand, recent measurements reveal that many existing licensed bands are widely underutilized. Cognitive radio technology has the great potential to alleviate spectrum scarcity by allowing cognitive radio devices to opportunistically access underutilized licensed spectrum while protecting the performance of licence holders [1].

Channel opportunities of cognitive radio users depend upon the activities of the license holders. When a license holder is inactive, its channel is available to the cognitive radio users. When the license holder is active, cognitive radio users must either cease transmission in that channel (in a spectrum overlay mode), or insure the amount of generated interference to the licence holder is below a certain threshold (in a spectrum underlay mode).

To utilize channel opportunities, cognitive radio users need *spectrum mobility* [1]. In other words, cognitive radio users need the ability to switch channels quickly to avoid significant interference with licence holders, to make use of spectrum opportunities, and to reduce the amount of interference they

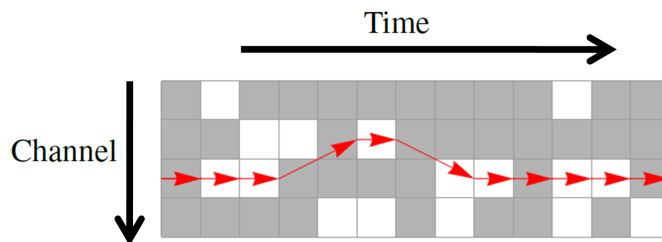


Fig. 1. An illustration of the frequency-time space. Each white (gray) block represents a time slot when a channel is available (not available). A user has prior knowledge of these channel availabilities, and plans when and how to switch channels (a sample plan is represented by a route connected by red arrows).

cause to each other¹. Ideally, cognitive radio users should be able to switch channels without causing any disruption to their current transmissions. However, in practice switching channels will inevitably cause disruption, in the forms of delay, potential packet loss, and the possibility of a broken transmission [4]. Therefore, cognitive radio users must balance the benefit and the cost of switching channels.

Channel switching has been extensively studied in cognitive radio networks, particularly in cases where users have no prior knowledge and channel availabilities change stochastically [10]. The problems in those scenarios turn out to be quite complex and often do not have closed-form solutions. In this paper, we consider the scenario where users have prior knowledge about the availabilities of multiple channels, over the next T time slots (i.e., they are given information about which frequency-time blocks will be available). Given this information, cognitive radio users need to formulate a *spectrum mobility plan* about when and how they should switch channels, to maximize their quality of service (see Fig. 1). A spectrum mobility plan corresponds to a route through frequency-time. The horizontal axis represents time (divided into T discrete time slots), whilst the vertical axis represents frequency (divided into discrete channels). Switching channels corresponds to a diagonal movement through frequency-time.

The question we wish to answer is *when and how should a user switch channels?* In particular, if the channel availability is changing rapidly, when is it worth the cost of switching channels, instead of simply waiting until the current channel becomes available again? This decision problem is nontrivial for a single cognitive radio user, and even more challenging

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¹The act of switching channels is also known as *spectrum handoff*.

when multiple cognitive radios compete to access the same set of available channels.

The motivation for studying channel switching using prior knowledge is as follows. Consider the TV white space, which is one of the most important licensed bands made accessible to cognitive radio users. The FCC has ruled that users must be given access to a predictive database which details the times when the license holders will be active/inactive over the next 24 hours [5]. Having this information alleviates the need for frequent sensing and reduces the probability that license holders suffer disruption from erroneous users. Our model allows us to characterize various properties of user behavior in such networks, all as functions of the switching cost.

We acknowledge that, in the TV white space scenario, channel availability changes at a large time scale, and so the cost of switching channels becomes negligible in comparison to the benefits of doing so. However, there are many scenarios, such as cellular networks and radars, where channel availabilities are much more dynamic [6]. Note that we do not advocate that third-party unlicensed users should access cellular spectrum. Instead, cognitive radio technologies can be applied to scenarios where macro-cells (high priority users) and femto-cells (low priority users) of the same service provider share spectrum dynamically. Because both macro and femto-cells belong to the same service provider and are both connected to the cellular backbone, there are clear incentives and means for macro-cells to share spectrum availability data with femto-cells to improve the overall efficiency and user experience. Furthermore, in a part of the 5GHz band, unlicensed WiFi devices are required to scan for radar signals and yield to radar transmissions. Radar scanning is semi-periodic and highly sensitive to interference. Here, channel availability information is readily available and can significantly benefit the protection of radar performance, and improve cognitive radio user's performance. Although these networks do not (yet) include prior knowledge on channel availability, it seems likely that such information will be made available in the future for the same reasons that motivated the FCC's recent ruling on TV white space (e.g., better spectrum access efficiency, lower overhead, and better licence holder performance protection). For example, consider a macro-cell serving voice, video streaming, and data users. Encoded voice traffic is semi-periodic. Video streaming traffic and data traffic can tolerate hundreds of milliseconds of delay. Therefore, the macro-cell base station can plan its transmissions and announce the schedule ahead of time so that femto-cells can act accordingly.

The problem of optimizing a spectrum mobility scheme based on prior knowledge of channel availabilities is complicated. On one hand, users wish to switch channels so that they can utilize spectrum opportunities and avoid interfering with each other. On the other hand, users wish to avoid the quality of service disruption that comes from switching channels too often. The optimal spectrum mobility plan for a single user can be determined using a shortest path algorithm. The multiple user case is essentially a game (see Section II). The strategies correspond to spectrum mobility plans. The payoff a user gets

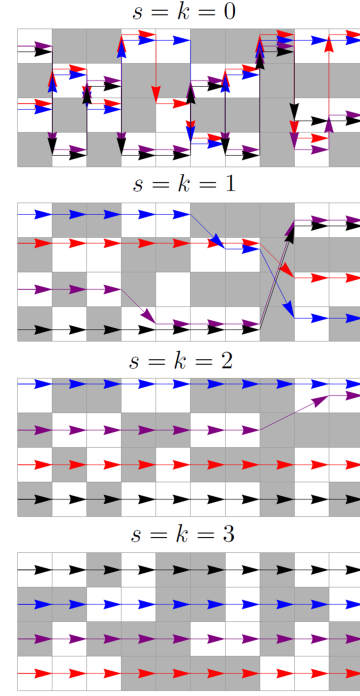


Fig. 2. Spectrum mobility games with 4 users. When a user visits an available channel (denoted by a white frequency time-block), it gains a payoff which decreases with the congestion level (i.e., the number of users on that channel). We show Nash equilibria for four versions of the game with different values of the switching time s and the switching cost k .

depends upon the quality of visited channels, the number of times they switch, and the amount of interference they suffer (see Fig. 2 for example).

The central result of this paper is that the spectrum mobility game is equivalent to a symmetric network congestion game (see Section III for detailed definitions). This result is very useful in practice, because symmetric network congestion games have nice properties² which allow us to design fast protocols for optimizing spectrum mobility plans in a distributed fashion (see Section IV). In Section V we use our protocol to simulate spectrum mobility. In Section VI we discuss related work on spectrum mobility. Our contributions can be summarized as follows:

- *A general model of spectrum mobility planning:* We present a general game theoretic model of spectrum mobility with prior knowledge of heterogeneous channels. Our model is applicable to both spectrum underlay and spectrum overlay networks.
- *Efficient algorithms for spectrum mobility planning:* We provide a fast algorithm to determine the best single-user decision, as a function of the other user's plans (Corollary 3). We also provide a decentralized protocol which allows multiple users to organize themselves into Nash equilibria without communicating with each other.
- *Analysis of behavior:* We use simulations to study the behavior of players at Nash equilibria of spectrum mobility

²Congestion games have the finite improvement property, meaning that if the players keep improving their strategy choices asynchronously, then a Nash equilibrium will eventually be reached.

games.

- *Application to real data:* We evaluate the performance of the proposed spectrum mobility planning algorithms using real channel availability data.

Due to space limitations, full proofs are given in the online technical report [12].

II. THE SPECTRUM MOBILITY GAME MODEL

A. Users, Channels, Time Slots, and Database

In this section we shall define the spectrum mobility game. The game involves a set $\mathcal{N} = \{1, 2, \dots, N\}$ of players, representing N cognitive radio users. There is a set $\mathcal{C} = \{1, 2, \dots, C\}$ of C heterogeneous channels. Time is divided into discrete time slots. The length of a time slot can be chosen to be any value that is suitable for the application scenario. The users have prior knowledge of the channel availabilities, i.e., they can access a *database* that describes the availability of all channels over the next T time slots. We let $\mathcal{T} = \{1, 2, \dots, T\}$ denote the set of these time slots. By checking the database, users plan how to act over the coming T time slots.

B. Channel Quality Functions

The database shows the channel quality functions $f_{(c,t)}(x)$ for each frequency-time point (c, t) with channel $c \in \mathcal{C}$ and time $t \in \mathcal{T}$. This function presents a user's payoff for using channel c at time t , when there are a total of x users on channel c at time t . It is non-negative and non-increasing in x , reflecting that a user's benefit from using a channel may decrease with the *congestion level*.

Our model allows each frequency-time point (c, t) to have a *different* channel quality function. This flexibility allows us to model many practical scenarios. We can allow one channel to have a higher bandwidth than another, i.e., $f_{(c,t)}(x) > f_{(c',t)}(x)$. We can also allow the benefit of using a channel to change over time in a way which represents license holder dynamics. For example, in a spectrum overlay network, we could set $f_{(c,t)}(x) = 0$ for all x to represent that the license holder of channel c is active at time t , and the channel cannot be accessed by the users. In a spectrum underlay network, we may have $f_{(c,t)}(x) = 0$ for all $x > J$, which represents the case where the license holder of channel c will not tolerate more than J cognitive radio users transmitting concurrently on that channel.

C. Switching Time s and Switching Cost k

We assume that it takes a fixed number of $s \geq 0$ time slots for a user to switch channels. More precisely, if a user initiates switching at the end of time slot t , then it will land on the destination channel at the beginning of time slot $t + s + 1$. We suppose that users do not gain any benefit, or generate any interference, while they are in the middle of switching.

A user must also pay a cost of $k \geq 0$ every time it switches channels. This allows us to model several possible negative effects of switching:

- *Additional power consumption:* This applies to a network scenario where a user needs to establish the connection

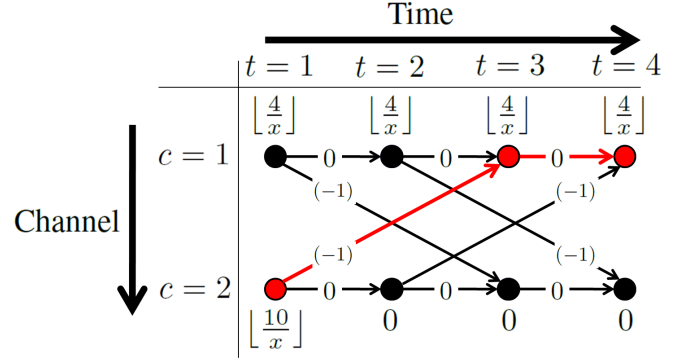


Fig. 3. Routes through the graph G^{mob} , representing different spectrum mobility plans available to a user. Vertices of G^{mob} represent frequency-time blocks (c, t) , and they are associated with channel quality functions $f_{(c,t)}(x)$. A zero function means that this channel is not available at this time. Users gain payoffs from visiting vertices. Users pay costs for traversing diagonal edges (which represent channel switching). In this case, we have $N = 1$ user, the switching cost is $k = 1$, and it takes $s = 1$ time slots to switch channels. The optimal spectrum mobility plan is shown in red (with thicker edges), and leads to a total payoff of $\lfloor \frac{10}{1} \rfloor - 1 + \lfloor \frac{4}{1} \rfloor + \lfloor \frac{4}{1} \rfloor = 17$.

to the new destination channel before tearing down the connection on the old channel.

- *Probability of connection failure:* This represents a small risk of not being able to establish the new connection due to various reasons.
- *Cost of sensing:* Although users have access to the database, a user may still need to engage in costly sensing to verify that its target channel is indeed available and/or to evaluate the channel quality and congestion level.

D. Graph Representation of Channel Switching

We model how users can switch channels with a directed graph³ $G^{\text{mob}} = (V^{\text{mob}}, E^{\text{mob}})$. Routes through the graph G^{mob} represent routes through frequency-time (see Fig. 3). When a user travels along a route, it gains payoffs at the vertices (which represent frequency-time blocks), and pays costs at the edges (which represent switching channels -at a cost of k , and staying on the same channel -at a cost of zero).

The graph $G^{\text{mob}} = (V^{\text{mob}}, E^{\text{mob}})$ is defined as follows:

- The vertex set is $V^{\text{mob}} = \mathcal{C} \times \mathcal{T}$. Vertex $(c, t) \in V^{\text{mob}}$ represents channel c in time slot t , i.e., a frequency-time block.
- The edge set is $E^{\text{mob}} = E_{\text{stick}}^{\text{mob}} \cup E_{\text{switch}}^{\text{mob}}$. The set $E_{\text{stick}}^{\text{mob}}$ represents the actions where users *do not switch channels*. Here $E_{\text{stick}}^{\text{mob}}$ is the set of all edges of the form $((c, t), (c, t+1))$ such that $c \in \mathcal{C}$ and $t \in \{1, 2, \dots, T-1\}$. The set $E_{\text{switch}}^{\text{mob}}$ represents the actions where users *switch channels*, i.e., the set of all edges of the form $((c, t), (c', t+s+1))$ such that $c, c' \in \mathcal{C} : c \neq c'$ and $t \in \{1, 2, \dots, T-1-s\}$.

In the red route in Fig. 3, the user switches from channel 2 to channel 1 at the end of time slot 1. This corresponds to traveling along the edge $((2, 1), (1, 3)) \in E_{\text{switch}}^{\text{mob}}$. The user arrives on channel 1 at the beginning of time slot 3 (and

³In a directed graph $G = (V, E)$, a vertex $u \in V$ is said to be connected to a vertex $v \in V$ if and only if (u, v) belongs to the edge set E .

effectively wastes $s = 1$ time slot switching). After this, the user sticks on channel 1. This corresponds to traveling along the edge $((1, 3), (1, 4)) \in E_{\text{stick}}^{\text{mob}}$.

A **route** r in a graph is a sequence of connected vertices (i.e., each vertex is connected to its successor in the sequence). We let $\mathcal{V}(r)$ denote the set of vertices traversed by the route r (including source and destination vertices), and let $\mathcal{E}(r)$ denote the set of edges traversed by the route r . In Fig. 3, for example, the route $r = ((2, 1), (1, 3), (1, 4))$ has vertices $\mathcal{V}(r) = \{(2, 1), (1, 3), (1, 4)\}$ and edges $\mathcal{E}(r) = \{((2, 1), (1, 3)), ((1, 3), (1, 4))\}$.

E. Spectrum Mobility Game

A **spectrum mobility game** is specified by a 6-tuple $\Gamma^{\text{mob}} = (\mathcal{N}, \mathcal{C}, \mathcal{T}, (f_{(c,t)})_{(c,t) \in \mathcal{C} \times \mathcal{T}}, s, k)$, where \mathcal{N} is the set of players, \mathcal{C} is the set of channels, \mathcal{T} is the set of time slots, $f_{(c,t)}$ is the (non-negative, non-increasing) function describing the quality of channel c at time t , s is the number of time slots required for a user to switch, and k is the switching cost. The information included in the 6-tuple can be used to construct the graph G^{mob} . Next we will explain how the players choose their strategies (routes across the graph) to maximize their payoffs.

Let \mathcal{R}^{mob} denote the set of all routes through G^{mob} , where each route goes from a vertex $(c, 1)$ with $c \in \mathcal{C}$ to a vertex (c', T) with $c' \in \mathcal{C}$. Here c can be the same as c' . Here \mathcal{R}^{mob} is the strategy set of each player, as each player (user) $n \in \mathcal{N}$ selects a route in \mathcal{R}^{mob} in the spectrum mobility game.

In our game, we suppose that every player selects its strategy (route) from \mathcal{R}^{mob} before the first time slot, i.e., a route in \mathcal{R}^{mob} is a **spectrum mobility plan** and does not change after the user actually starts to move through frequency-time. In fact, we can show that the users will not change their plan after the first time slot even if they are allowed to do so, as users know the complete network information at the beginning of the game already.

Players accumulate payoffs for traversing vertices in their routes, while paying costs for traversing edges. The payoff a player gets from traversing a vertex $v = (c, t) \in V^{\text{mob}}$ is equal to its channel quality function $f_{(c,t)}(x)$, where x is the total number of players which select routes that traverse this vertex. In other words, x is the total number of users of channel c at time t . Each edge $e \in E^{\text{mob}}$ is associated with a fixed cost $\text{cost}(e)$. If $e \in E_{\text{stick}}^{\text{mob}}$, then $\text{cost}(e) = 0$ because sticking upon a channel does not cost anything. If $e \in E_{\text{switch}}^{\text{mob}}$, then $\text{cost}(e) = k$ because a user must pay a cost of k every time it switches channels.

In the spectrum mobility game, each player $n \in \mathcal{N}$ selects a route $X_n \in \mathcal{R}^{\text{mob}}$. A **strategy profile** $\mathbf{X} = (X_{n'})_{n' \in \mathcal{N}} \in (\mathcal{R}^{\text{mob}})^N$ consists of each player's choice of route. The **total payoff** received by a player n within the strategy profile \mathbf{X} is given by

$$\text{PAY}_n^{\text{mob}}(\mathbf{X}) = \sum_{v \in \mathcal{V}(X_n)} f_v(\psi_{\mathbf{X}}(v)) - \sum_{e \in \mathcal{E}(X_n)} \text{cost}(e), \quad (1)$$

where $\psi_{\mathbf{X}}(v) = |\{n' \in \mathcal{N} : v \in \mathcal{V}(X_{n'})\}|$ is the total number

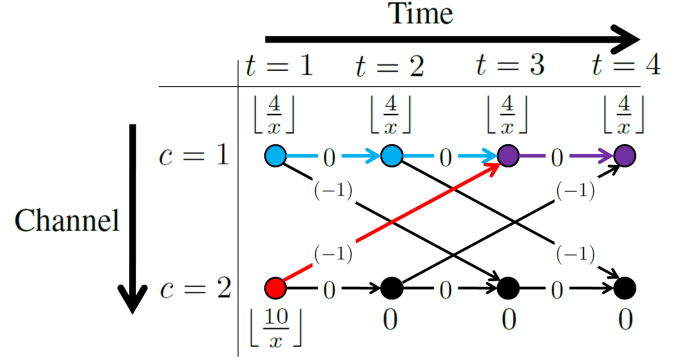


Fig. 4. This spectrum mobility game is identical to the one depicted in Fig. 3, except we have $N = 2$ users. The red user's strategy is the same as in Fig. 3, while the blue user's strategy consists of staying on channel 1 for all of the $T = 4$ time slots. The edges and vertices shared by the two users at the Nash equilibrium are colored purple. The blue user will gain a payoff of 4 upon time slot 1 and time slot 2, will only gain a payoff of 2 upon time slots 3 and 4 (because it has to share this channel with the newly arrived red user). In this Nash equilibrium, the red user will gain a total payoff of $f_{(2,1)}(1) - 1 + f_{(1,3)}(2) + f_{(1,4)}(2) = 10 - 1 + 2 + 2$, while the blue user will gain a total payoff of $f_{(1,1)}(1) + f_{(1,2)}(1) + f_{(1,3)}(2) + f_{(1,4)}(2) = 4 + 4 + 2 + 2$.

of users whose chosen routes visit the vertex v . The first sum in (1) represents the payoff that user n gains from transmitting on channels. The second sum in (1) represents the cost it pays due to switching.

A **Nash equilibrium**⁴ of a game is a strategy profile where no player can benefit from changing its strategy unilaterally. See Fig. 4 for such an example. In the next section, we will show that every spectrum mobility game is equivalent to a *symmetric network congestion game*. This interesting and highly non-trivial result allows us to develop a fast protocol to find Nash equilibria of spectrum mobility game.

III. THE CORRESPONDENCE BETWEEN SPECTRUM MOBILITY GAMES AND NETWORK CONGESTION GAMES

A. Definition of Symmetric Network Congestion Games

Congestion games [13] have been used to study many network resource allocation scenarios. The basic idea behind congestion games is that a player needs to pay a cost $d_e(x)$ when he uses a resource e with a congestion level x . In a *network congestion game* [14], the resources are edges within a graph. Each player selects a route from their source vertex to their destination vertex, in an attempt to minimize the cost they pay from traversing congested edges. For example, network congestion games can model how drivers select routes through cities. A network congestion game is *symmetric* [15] when every player selects a route from the same source vertex to the same destination vertex.

Formally, a **symmetric network congestion game** $(\mathcal{N}, G, a, b, (d_e)_{e \in E})$ consists of the following:

- A set $\mathcal{N} = \{1, 2, \dots, N\}$ of players.
- A directed graph $G = (V, E)$.
- A common source vertex $a \in V$ and a common destination vertex $b \in V$.

⁴In this paper we only discuss pure strategies and pure Nash equilibria.

- A non-increasing and non-negative cost function $d_e(\cdot)$ for each edge $e \in E$.

A strategy in a symmetric network congestion game is a route from a to b in G . The strategy set of each player is equal to the set \mathcal{R}^* of all routes from a to b in G . A strategy profile $\mathbf{X} = (X_{n'})_{n' \in \mathcal{N}} \in (\mathcal{R}^*)^N$ involves each player n' choosing a route $X_{n'} \in \mathcal{R}^*$. The total cost to a player n in a strategy profile \mathbf{X} is

$$\text{COST}_n(\mathbf{X}) = \sum_{e \in \mathcal{E}(X_n)} d_e(\psi_{\mathbf{X}}^*(e)), \quad (2)$$

where $\mathcal{E}(r)$ is the set of edges in route r and $\psi_{\mathbf{X}}^*(e) = |\{n' \in \mathcal{N} : e \in \mathcal{E}(X_{n'})\}|$ is the total number of players using edge e .

Since symmetric network congestion games are congestion games, they have the **finite improvement property** [16]. This means that when players keep *improving* their strategy choices asynchronously where no more than one player changes its strategy at any given time, the system will eventually reach a Nash equilibrium.

B. Correspondence with Spectrum Mobility Games

Spectrum mobility games and symmetric network congestion games are similar. They both involve players selecting routes through a graph structure in an attempt to avoid congestion from one another. However, the two types of games differ in several ways:

- 1) In a symmetric network congestion game, each player tries to minimize its total cost function $\text{COST}_n(\mathbf{X})$; whereas in the spectrum mobility game, each player tries to maximize its payoff function $\text{PAY}_n^{\text{mob}}(\mathbf{X})$.
- 2) In a symmetric network congestion game, a player's cost only depends on the edges belonging to its chosen route; whereas in the spectrum mobility game, a player's payoff depends on both the edges and the vertices belonging to its chosen route.
- 3) In a symmetric network congestion game, each player selects a route which goes from the same source to the same destination; whereas in the spectrum mobility game, the players may pick different starting points $(c, 1)$ and end points (c', T) .

On the other hand, we can convert a spectrum mobility game into a symmetric network congestion game by making the following three alterations to the spectrum mobility game (note how these undo the respective differences listed above):

- 1) *Convert payoff maximization into cost minimization:* We transform the spectrum mobility game (within which players wish to maximize payoffs) into a regret minimization game. In any given time slot, a player's regret is equal to the maximum possible payoff⁵ $Q = \max\{f_{(c,t)}(1) : c \in \mathcal{C}, t \in \mathcal{T}\}$ minus the payoff gained in the spectrum mobility game. For example, usage of channel c at time t leads to a payoff gain of $f_{(c,t)}(x)$, hence we associate it with a regret function of

⁵ Q is chosen to be large enough such that the regret is always nonnegative.

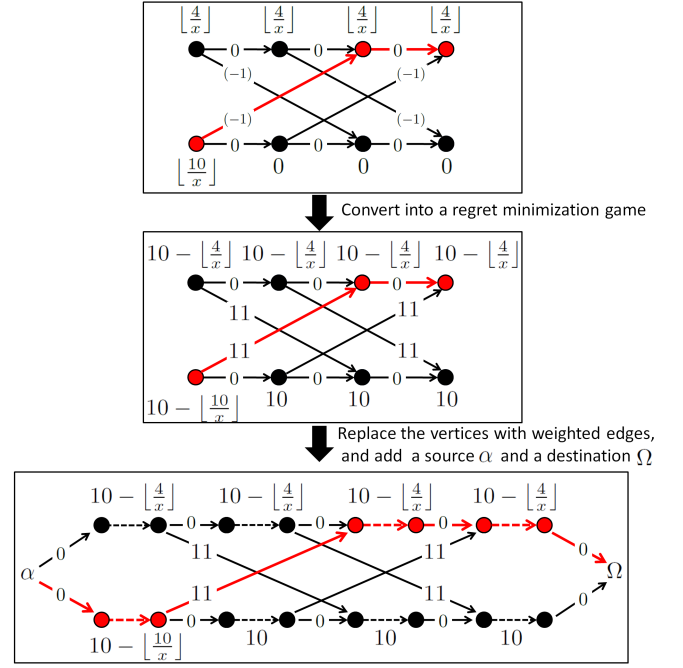


Fig. 5. The spectrum mobility game shown in Fig. 3 and Fig. 4 can be converted into a symmetric network congestion game by making 3 alterations. Here $Q = 10$.

$Q - f_{(c,t)}(x)$. Switching channels leads to a payoff gain of $-k$ and takes s time slots, hence we associate it with a regret function of $sQ + k$. The total regret incurred by a player n over T time slots is $TQ - \text{PAY}_n^{\text{mob}}(\mathbf{X})$, and is equal to the cost in the corresponding symmetric network congestion game. This regret minimization game is equivalent to the spectrum mobility game.

- 2) *Replace vertices with edges:* Replace every vertex in the spectrum mobility game with an edge which is associated with the same cost/regret function as the old (replaced) vertex.
- 3) *Add virtual source and destination vertices:* Add a virtual source vertex α and a virtual destination vertex Ω , which are connected to all the starting points and ending points (respectively) with zero cost edges.

Figure 5 illustrates how the three alterations transform a spectrum mobility game into a symmetric network congestion game. Mathematically, we can state the following results.

Theorem 1: Every spectrum mobility game Γ^{mob} is equivalent⁶ to some symmetric network congestion game Γ^{net} .

For the proof sketch, see Appendix A. For a detailed proof, see the technical report [12].

Corollary 2: Every spectrum mobility game has the finite improvement property, and hence has at least one pure Nash equilibrium.

The **best response** $B_n^{\Gamma^{\text{mob}}}(\mathbf{X}_{-n})$ of a player n within the strategy profile \mathbf{X} of the spectrum mobility game Γ^{mob} is the strategy which maximizes n 's payoff, given the list

⁶We say two games are *equivalent* when there is a weak isomorphism from one to the other [17]. Such a mapping preserves the payoff orderings of the strategies, the finite improvement property, and the Nash equilibria.

$\mathbf{X}_{-n} = (X_1, \dots, X_{n-1}, X_{n+1}, \dots, X_N)$ of strategies of the other players in strategy profile \mathbf{X} . The best response can be a set with many elements in general, but we assume there is some deterministic tie breaking mechanism so $B_n^{\Gamma^{\text{mob}}}(\mathbf{X}_{-n})$ is single valued. Corollary 2 implies that if players keep asynchronously updating their strategies according to their best responses, then a Nash equilibrium will eventually be reached (see Fig. 6).

Corollary 3: A player in the spectrum mobility game can determine its best response route choice $B_n^{\Gamma^{\text{mob}}}(\mathbf{X}_{-n})$ within polynomial time by using a shortest path finding algorithm.

For the proof sketch, see Appendix B. For a detailed proof, see the technical report [12]. Corollary 3 implies that multiple users can quickly adapt to changes in each others' plans.

IV. PROTOCOL TO FIND NASH EQUILIBRIA

The correspondence between spectrum mobility games and symmetric network congestion games allows us to design a spectrum mobility planning protocol. The purpose of this protocol is to allow the users to select their routes through frequency-time in a mutually acceptable and efficient way, i.e., reaching a Nash equilibrium.

Our protocol is based on the finite improvement property shown in Corollary 2. The key idea is to let players asynchronously improve their spectrum mobility plans (i.e., routes across the graph) until a Nash equilibrium is reached. We will first present a *deterministic* version of the protocol, with an illustrative example shown in Fig. 6. Later we shall discuss how the protocol can be adapted to various scenarios.

Suppose we have a spectrum mobility game $\Gamma^{\text{mob}} = (\mathcal{N}, \mathcal{C}, \mathcal{T}, (f_{(c,t)}(c,t) \in \mathcal{C} \times \mathcal{T}, s, k))$. We assume that players are indexed in a way that is publicly known,⁷ and have the ability to communicate their spectrum mobility plans to each other. We write $\mathbf{X} = (X_1, X_2, \dots, X_N)$ as the list of current the spectrum mobility plans selected by the players. This gets updated as the protocol runs.

Algorithm 1 is the spectrum mobility planning protocol. It has two phases. During the initialization phase, each player chooses the strategy $B_0^{\Gamma^{\text{mob}}}(\emptyset)$, which is the best response assuming that they are the only user in the network (see Fig. 1). In other words, $B_0^{\Gamma^{\text{mob}}}(\emptyset)$ would maximize a player's total payoff if the other players did not exist. The initial route choices of

⁷For example, the users' indices could correspond to the order in which they entered the network or their MAC addresses.

Algorithm 1: Spectrum Mobility Planning Protocol

Input: A spectrum mobility game Γ^{mob}

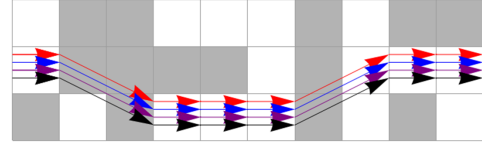
Output: A Nash equilibrium \mathbf{X} of Γ^{mob}

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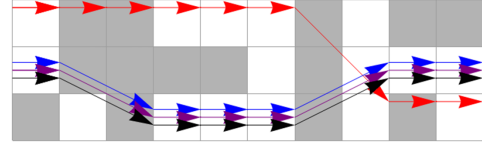
1 for  $n = 1$  to  $N$  do
2    $X_n \leftarrow B_0^{\Gamma^{\text{mob}}}(\emptyset)$ ;
3 while  $\mathbf{X}$  is not a Nash equilibrium do
4   for  $n = 1$  to  $N$  do
5      $X_n \leftarrow B_n^{\Gamma^{\text{mob}}}(\mathbf{X}_{-n})$ ;
6 return  $\mathbf{X}$ ;

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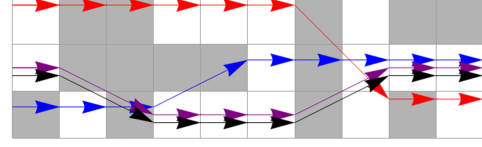
All players start using the same strategy



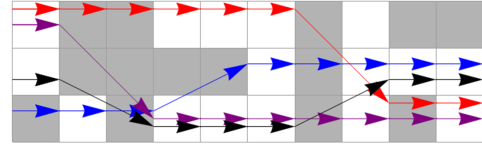
Player 1 updates to the best response



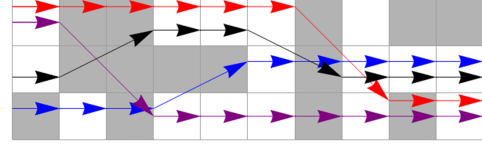
Player 2 updates to the best response



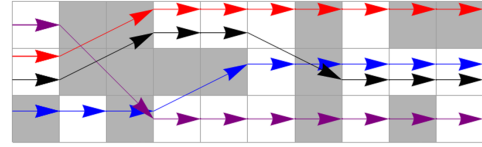
Player 3 updates to the best response



Player 4 updates to the best response



Player 1 updates to the best response



Nash equilibrium reached

Fig. 6. An illustration of our spectrum mobility protocol being used to generate a Nash equilibrium. Initially each of the four players selects the same route through frequency-time, then the players do asynchronous best response updates (in the order 1, 2, 3, 4, 1, 2, 3, ...) until a Nash equilibrium is reached. In this case the system reaches a Nash equilibrium after 5 updates.

players are not essential; players can select any routes and the protocol will still converge eventually. In the second iterative updating phase, each player updates its strategy to the best response $X_n = B_n^{\Gamma^{\text{mob}}}(\mathbf{X}_{-n})$, and the finite improvement property guarantees that a Nash equilibrium will be achieved.

One can use similar ideas to make an alternative random protocol, by letting the players start with random strategies X_n , and having them perform best response updates in a random and asynchronous order. This could be more suitable in networks within which users do not acknowledge a common ordering. To implement this random protocol, users need

to pass messages to each other through a common control channel.

Comparing with a random protocol, one major advantage of the deterministic protocol is that it actually *does not* require users to communicate. As the output of the protocol in Algorithm 1 is deterministic, each user could run the protocol locally, compute the same Nash equilibrium, and then choose the corresponding routes accordingly.

To implement the deterministic approach, users need to know the initial state of each user and a commonly agreed user ordering. In networks where the set of users remains constant (e.g., wireless sensor networks), the users ordering could be fixed initially. In dynamic scenarios, users would need to keep track of when each other enter and leave the system in order to maintain a common ordering. Alternatively, users could conservatively assume that all users in their vicinity are active. This would lead to inefficiency, because users would be trying to avoid incurring interference from absent phantom users in order to avoid explicit communications. The user's actions under such assumptions may not correspond to Nash equilibria.

One limitation of our protocol is that it is not (theoretically) guaranteed to converge within polynomial time (although in our simulations the protocol does converge very quickly, with run time increasing quadratically in N, C , and T). On the other hand, when the channel quality functions (and switching costs) of the spectrum mobility game are integer valued, the game corresponds to a symmetric network congestion game with integer valued cost functions. There is a polynomial time algorithm (described in [15]) which can be used to find a Nash equilibrium of such symmetric network congestion games. We provide more details about this observation, and other protocols in the technical report [12].

V. SIMULATIONS

We applied our proposed models to the study of spectrum mobility using real channel availability data. The data we used (from [18]) was a record of the availabilities of $C = 3$ channels (850-870MHz band) over a total length of 1 minute in Maryland. The time is divided into $T = 600$ time slots that are 0.1 seconds long each. The data can be represented by a 3×600 binary matrix D^* such that

$$D_{c,t}^* = \begin{cases} 1, & \text{if channel } c \text{ is available on time slot } t \\ 0, & \text{otherwise.} \end{cases}$$

The detailed data trace can be found in our online technical report [12]. We use the database D^* to set up a spectrum mobility game within which each frequency-time block (c, t) has a channel quality function $f_{(c,t)}^*(x) = D_{c,t}^*/x$.

We studied the behavior of $N = 10$ users at the Nash equilibria (computed using our spectrum mobility protocol). We consider a switching cost $k = 0$, and investigate the effect of the switching time s (see Figs. 7, 8, and 9). Out of the 3×600 frequency-time blocks, only 1173 are available. In each Nash equilibrium computed, at least one player accesses each available frequency-time block; because of the form of

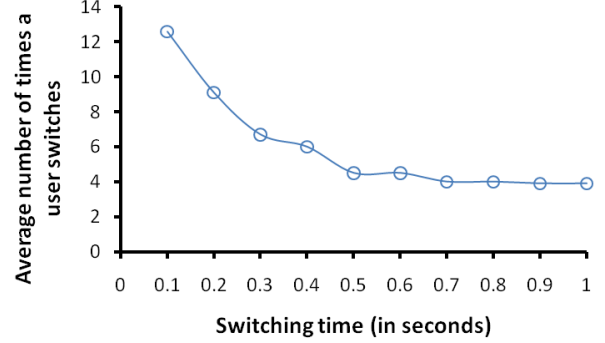


Fig. 7. At the Nash equilibria of the spectrum mobility game, the average number of times that a user switches decreases with the switching time s .

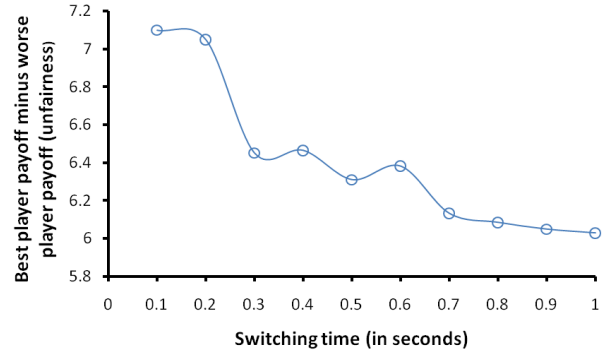


Fig. 8. At the Nash equilibria of the spectrum mobility game, the fairness among users tends to increase with the switching time s .

the channel quality functions $f_{(c,t)}^*$, the average payoff of the players is thus equal to 117.3 in all cases. Figure 7 shows that the average number of switching per user decreases as the switching time s increases. Figure 8 suggests that the Nash equilibria produced by our algorithm are fairer when the switching time is larger. One possible explanation is that when switching time is smaller, the first players to select their strategies within our best response based algorithm have more freedom to secure spectrum opportunities than the players which formulate their plans later. Figure 9 shows that the average congestion level of a block decreases with the switching time s . This is because when s is larger, more users spend a longer time switching channels, and thus cause less congestions to users who are not switching.

VI. RELATED WORK

Most studies of spectrum mobility have focused on the stochastic channel models. The main challenge in this line of work is to learn the state of the channel availability and balance among multiple users. For example, in [10], [19], [25], the authors study the multi-channel probing and access problems. The studies involve partially Observable Markov Decision Process (POMDP) or multi-armed bandit methods. In [20], the authors characterized the tradeoff between maximizing the total throughput of licence holders and cognitive radio users and minimizing licence holders' interference.

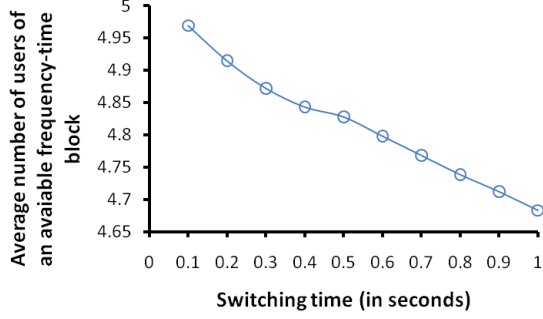


Fig. 9. At the Nash equilibria of the spectrum mobility game, the average number of users of an available frequency-time block decreases with the switching time s .

The stochastic scenarios are complex with many factors to consider, and thus it is in general difficult to get closed-form solutions and clean engineering insights. Thus many results focus upon evaluating the efficiency of spectrum mobility schemes under more simplified channel availability models. The authors in [7] and [8] use a two-dimensional Markov channel availability model, and do not distinguish between different channels. In [9], the authors consider spectrum mobility in a network with two users. In [11], the efficiency of two spectrum mobility schemes are compared using queuing theory. In our paper, because of the assumption that channel availabilities are known in advance, we can analyze the system performance precisely under a wide range of assumptions of channel conditions, switching costs, and the number of users.

Centralized and distributed schemes for spectrum allocations or multi-channel dynamic spectrum access have been considered in many recent papers (e.g., [2], [3], [10], [19], [21]–[23], [25]–[31]). Our work differs because we consider complete prior knowledge of channel availability. Complete knowledge provides much better primary user protection as well as overall spectrum efficiency. It also provides an upper bound on the performance of systems with stochastic channel availabilities. Also we explicitly model the competition of multiple users and formulate it as game. To the best of our knowledge, this is the first work formulating the problem as a game under the assumption of complete prior knowledge of channel availability.

VII. CONCLUSION

Our main contribution is the formulation of the spectrum mobility game, which is a very general model of spectrum mobility planing. A key step of the analysis is the non-trivial conversion from the spectrum mobility game to the symmetric network congestion game. At the first glance, the problem of determining the optimum spectrum mobility plan of a single user looks like a longest path problem, where one attempts to maximize total quality of the frequency-time blocks visited. Our key observation is that this can be converted into a regret minimization problem, which essentially corresponds to a shortest path problem with *non-negative* edge weights. This is the crucial step within the conversion, which naturally

maps the spectrum mobility game into a symmetric network congestion game in the case with multiple users.

The above conversion has allowed us to use tools from the theory of graphs and congestion games to develop useful spectrum mobility protocols. In particular, we showed that there exists a polynomial time protocol which can be used to determine the optimal policy in the one user case, and characterize Nash equilibria in the multiple user case when the payoff functions are integer valued.

The converting method proposed in this paper can also be applied to other scenarios with spatial reuse and heterogeneous users (who have different tastes for the same channel). A scenario with spatial reuse may be translated into a *congestion game on a graph* similar to those considered in [13], [32]. User heterogeneity can be accounted for by making the payoff functions player specific. It will be interesting to study the conditions under which a pure Nash equilibrium exists, as the existence is not guaranteed in generalized network congestion games that result from such a conversion [32], [33].

In future work we shall study how the amount of prior knowledge a user affects its performance. We also intend to extend our models from deterministic cases to stochastic cases where there is some uncertainty about the future availability of channels. Stochastic games seem to be appropriate models for these scenarios. This generalization will be challenging, but should be applicable to an even wider range of cognitive radio scenarios where the users are given limited information about the future. We also intend to extend the idea of modeling sequential game choices using graph paths to other game theoretic scenarios.

Due to space limitations, we have included the full proofs of all results in this paper as well as more algorithms and data in the online technical report [12].

APPENDIX A PROOF SKETCH OF THEOREM 1

Sketch of proof We describe the main points of our constructive proof here. Our equivalent network congestion game $\Gamma^{\text{net}} = (\mathcal{N}, G^{\text{net}}, \alpha, \Omega, (d_e^{\text{net}})_{e \in E^{\text{net}}})$ is played upon the directed graph $G^{\text{net}} = (V^{\text{net}}, E^{\text{net}})$. We will show routes through G^{net} correspond to strategies within Γ^{mob} . The graph G^{net} has vertex set $V^{\text{net}} = \{\alpha, \Omega\} \cup (\mathcal{C} \times \mathcal{T} \times \{0, 1\})$ and edge set $E^{\text{net}} = E_{\alpha}^{\text{net}} \cup E_0^{\text{net}} \cup E_{\text{stick}}^{\text{net}} \cup E_{\text{switch}}^{\text{net}} \cup E_{\Omega}^{\text{net}}$, where $E_{\alpha}^{\text{net}} = \{(\alpha, (c, 1, 0)) : c \in \mathcal{C}\}$, $E_0^{\text{net}} = \{((c, t, 0), (c, t, 1)) : c \in \mathcal{C}, t \in \mathcal{T}\}$, $E_{\text{stick}}^{\text{net}} = \{((c, t, 1), (c, t+1, 0)) : c \in \mathcal{C}, t \in \{1, 2, \dots, T-1\}\}$, $E_{\text{switch}}^{\text{net}} = \{((c, t, 1), (c', t+s+1, 0)) : c, c' \in \mathcal{C}, c \neq c', t \in \{1, 2, \dots, T-1-s\}\}$, and $E_{\Omega}^{\text{net}} = \{((c, T, 1), \Omega) : c \in \mathcal{C}\}$. We associate each edge $e \in E^{\text{net}}$ with a non-decreasing and non-negative cost function d_e^{net} , which is defined as follows:

- If $e \in E_{\alpha}^{\text{net}} \cup E_{\text{stick}}^{\text{net}} \cup E_{\Omega}^{\text{net}}$, then $d_e^{\text{net}}(x) = 0, \forall x$.
- If $e \in E_{\text{switch}}^{\text{net}}$, then $d_e^{\text{net}}(x) = s \cdot Q + k, \forall x$.
- If $e \in E_0^{\text{net}}$, then e must be of the form $e = ((c, t, 0), (c, t, 1))$ and so we let $d_e^{\text{net}}(x) = Q - f_{(c,t)}(x), \forall x$.

Here $Q = \max\{f_{(c,t)}(1) : c \in \mathcal{C}, t \in \mathcal{T}\}$. Let \mathcal{R}^{net} denote the set of all routes r from α to Ω in G^{net} . Now \mathcal{R}^{net} is the strategy set of each player $n \in \mathcal{N}$, in the symmetric network congestion game Γ^{net} . The total cost incurred by a player $n \in \mathcal{N}$ for playing the strategy $Y_n \in \mathcal{R}^{\text{net}}$ within the strategy profile \mathbf{Y} is given by $\text{COST}_n^{\text{net}}(\mathbf{Y}) = \sum_{e \in \mathcal{E}(Y_n)} d_e^{\text{net}}(|\{n' \in \mathcal{N} : e \in \mathcal{E}(Y_{n'})\}|)$. Now, the bijection $I : \mathcal{R}^{\text{mob}} \mapsto \mathcal{R}^{\text{net}}$ which sends each route $r^* = ((c_1, t_1), (c_2, t_2), \dots, (c_m, t_m)) \in \mathcal{R}^{\text{mob}}$ (through G^{mob}) to the strategy $I(r^*) = (\alpha, (c_1, t_1, 0), (c_1, t_1, 1), (c_2, t_2, 0), (c_2, t_2, 1), \dots, (c_m, t_m, 0), (c_m, t_m, 1), \Omega) \in \mathcal{R}^{\text{net}}$ essentially converts Γ^{mob} into Γ^{net} . More precisely, if we consider that each player n in Γ^{mob} is trying to minimize a total cost function $\text{COST}_n^{\text{mob}}(\mathbf{X}) := -\text{PAY}_n^{\text{mob}}(\mathbf{X})$ then we have $\text{COST}_n^{\text{mob}}(\mathbf{X}) = \text{COST}_n^{\text{net}}(I(\mathbf{X}_{n'}))_{n' \in \mathcal{N}} - TQ, \forall n \in \mathcal{N}$, for each strategy profile $\mathbf{X} \in (\mathcal{R}^{\text{mob}})^{\mathcal{N}}$. Hence we have converted Γ^{mob} into Γ^{net} by using the [17] strong game isomorphism I and then a constant shift of the global cost functions by $-TQ$. \square

APPENDIX B PROOF SKETCH OF COROLLARY 3

Sketch of proof The best response of a player n in a strategy profile \mathbf{Y} of a symmetric network congestion game $(\mathcal{N}, G, a, b, (d_e)_{e \in E})$ is the shortest path from the source vertex a to the destination vertex b within the graph G' obtained by assigning a weight of $d_e(x+1)$ to each edge e of $G = (V, E)$. Here $x = |\{n' \in \mathcal{N} : n' \neq n, e \in \mathcal{E}(Y_{n'})\}|$ is the number of other players using edge e . This shortest path through G' (i.e., the best response) can be determined in polynomial time using Dijkstra's shortest path algorithm. Hence, given a strategy profile \mathbf{X} of Γ^{mob} , we can determine player n 's best response by applying the isomorphism I (defined in the proof to Theorem 1) to convert \mathbf{X} into the equivalent strategy profile $\mathbf{Y} = (I(\mathbf{X}_{n'}))_{n' \in \mathcal{N}}$ of the equivalent symmetric network congestion game Γ^{net} . We can then determine n 's best response r^* in Γ^{net} (using a shortest path algorithm), and now $I^{-1}(r^*)$ will be n 's best response in Γ^{mob} . \square

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